Assignment 1

1) Convert the following sentences to Conjunctive Normal Form (CNF)

a) (P → Q) → ((Q → R) → (P → R))

≡ (¬P ∨ Q) → ((Q → R) → (P → R))

≡ (¬P ∨ Q) → ((¬Q ∨ R) → (P → R))

≡ (¬P ∨ Q) → ((¬Q ∨ R) → (¬P ∨ R))

≡ (¬P ∨ Q) → (¬ (¬Q ∨ R) ∨ (¬P ∨ R))

≡ (¬P ∨ Q) → ((Q ∧ ¬R) ∨ (¬P ∨ R))

≡ ¬ (¬P ∨ Q) ∨ ((Q ∧ ¬R) ∨ (¬P ∨ R))

≡ (P ∧ ¬Q) ∨ ((Q ∧ ¬R) ∨ (¬P ∨ R))

≡ (P ∧ ¬Q) ∨ ((Q ∨ ¬P ∨ R) ∧ (¬R ∨ P V R))

≡ (P ∧ ¬Q) ∨ Q ∨ ¬P ∨ R

≡ (P ∨ Q ∨ ¬P ∨ R) ∧ (¬Q ∨ Q ∨ ¬P ∨ R)

≡ (1 ∨ Q ∨ R) ∧ (1 ∨ ¬P ∨ R)

≡ (1) ∧ (1)

≡ (1)

b) (P → Q) ↔ (P → R)

≡ (¬P ∨ Q) ↔ (¬P ∨ R)

≡ ((¬P ∨ Q) → (¬P ∨ R)) ∧ ((¬P ∨ R) → (¬P ∨ Q))

≡ (¬ (¬P ∨ Q) ∨ (¬P ∨ R)) ∧ (¬ (¬P ∨ R) ∨ (¬P ∨ Q))

≡ ((P ∧ ¬Q) ∨ (¬P ∨ R)) ∧ ((P ∧ ¬R) ∨ (¬P ∨ Q))

≡ ((P ∨ ¬P ∨ R) ∧ (¬Q ∨ ¬P ∨ R)) ∧ ((P ∨ ¬P ∨ Q) ∧ (¬R ∨ ¬P ∨ Q))

≡ (¬Q ∨ ¬P ∨ R) ∧ (¬R ∨ ¬P ∨ Q)

c) (P ∧ Q) → (¬P ↔ Q)

≡ (P ∧ Q) → ((¬P → Q) ∧ (Q → ¬P))

≡ (P ∧ Q) → ((P ∨ Q) ∧ (¬Q ∨ ¬P))

≡ ¬ (P ∧ Q) ∨ ((P ∨ Q) ∧ (¬Q ∨ ¬P))

≡ (¬P ∨ ¬Q) ∨ ((P ∨ Q) ∧ (¬Q ∨ ¬P))

≡ ((¬P ∨ ¬Q) ∨ (P ∨ Q)) ∧ ((¬P ∨ ¬Q) ∨ (¬Q ∨ ¬P))

≡ (¬P ∨ ¬Q ∨ P ∨ Q) ∧ (¬P ∨ ¬Q)

≡ (1) ∧ (¬P ∨ ¬Q)

2a.1) Translate the reasoning into propositional logic formulas. Use s, j, b, p for atomic propositions that are true if Sydney, Johnson, Benson, or Presley (respectively) have a dog, and write S, J, B, P for atomic propositions that are true if they have a cat.

F1: If Mr. Sydney has a dog, then Mrs. Benson has a cat.

F2: If Mr. Johnson has a dog, then he has a cat, too.

F3: If Mr. Sydney has a dog and Mr. Johnson has a cat, then Mrs. Presley has a dog.

F4: If Mrs. Benson and Mr. Johnson share a pet of the same species, then Mr. Sydney has a cat.

F5: Mr. Sydney and Mr. Johnson have dogs.

F1: s → B

F2: j → J

F3: (s ∧ J) → p

F4: ((b ∧ j) ∨ (B ∧ J)) → S

F5: s ∧ j

2a.2) Let F = F1 ∧ F2 ∧ F3 ∧ F4 ∧ F5. Check F for satisfiability using the Horn’s formula satisfiability test. If you verify that F is satisfiable, then present a model for it. Justify.

F = F1 ∧ F2 ∧ F3 ∧ F4 ∧ F5

≡ (s → B) ∧ (j → J) ∧ ((s ∧ J) → p) ∧ (((b ∧ j) ∨ (B ∧ J)) → S) ∧ (s ∧ j)

≡ (s → B) ∧ (j → J) ∧ ((s ∧ J) → p) ∧ (((b → ¬j) → (B ∧ J)) → S) ∧ (1→ s) ∧ (1 → j)

𝒜(s) = ~~0~~, 1

𝒜(j) = ~~0~~, 1

𝒜(b) = 0

𝒜(p) = ~~0~~, 1

𝒜(S) = ~~0~~, 1

𝒜(J) = ~~0~~, 1

𝒜(B) = ~~0~~, 1

Formula is satisfiable with the model 𝒜(b) = 𝒜(P) = 0 and 𝒜(s) = 𝒜(j) = 𝒜(p) = 𝒜(S) = 𝒜(J) = 𝒜(B) = 1.

2b) Check the following formula for satisfiability using the Horn’s formula satisfiability test. If you verify that the formula is satisfiable, then present a model for it.

(¬A ∨ E) ∧ ¬B ∧ (¬C ∨ (A →B)) ∧ A ∧ (¬E ∨ C ∨ ¬D) ∧ (D ∧ (D ∨ F))

≡ (A → E) ∧ (B → 0) ∧ (C → (A →B)) ∧ (1 → A) ∧ ((E ∧ D) → C) ∧ ((D ∧ D) ∨ (D ∧ F))

≡ (A → E) ∧ (B → 0) ∧ (C → (A →B)) ∧ (1 → A) ∧ ((E ∧ D) → C) ∧ (¬ (D) → (D ∧ F))

≡ (A → E) ∧ (B → 0) ∧ (C → (A →B)) ∧ (1 → A) ∧ ((E ∧ D) → C) ∧ (D → 0) → (D ∧ F))

𝒜(A) = ~~0~~, 1

𝒜(B) = 0

𝒜(C) = 0

𝒜(D) = 0

𝒜(E) = ~~0~~, 1

𝒜(F) = 0

Formula is satisfiable with the model 𝒜(B) = 𝒜(C) = 𝒜(D) = 𝒜(F) = 0 and 𝒜(A) = 𝒜(E) = 1.

3a) Prove or disprove the following claim: (P → Q) ∧ ((Q ∧ R) → S) ∧ ¬ (P → ¬R) ⊨ S using the unit resolution strategy.

F = (P → Q) ∧ ((Q ∧ R) → S) ∧ ¬ (P → ¬R) ⊨ S

≡ (¬P ∨ Q) ∧ (¬Q ∨ ¬R ∨ S) ∧ ¬ (¬P ∨ ¬R) ⊨ S

≡ (¬P ∨ Q) ∧ (¬Q ∨ ¬R ∨ S) ∧ (P ∧ R) ⊨ S

≡ (¬P ∨ Q) ∧ (¬Q ∨ ¬R ∨ S) ∧ P ∧ R ⊨ S

Set of Clauses:

F = {{¬P, Q}, {¬Q, ¬R, S}, {P}, {R}, {¬S}}

F = {{¬P, Q}, {¬Q, ¬R, S}, {P}, {R}, {¬S}, {Q}} Clause 1 and 3

F = {{¬P, Q}, {¬Q, ¬R, S}, {P}, {R}, {¬S}, {Q}, {¬R, S}} Clause 2 and 6

F = {{¬P, Q}, {¬Q, ¬R, S}, {P}, {R}, {¬S}, {Q}, {¬R, S}, {S}} Clause 4 and 7

F = {{¬P, Q}, {¬Q, ¬R, S}, {P}, {R}, {¬S}, {Q}, {¬R, S}, {S}, □} Clause 5 and 8

Since the output is the empty clause and F is unsatisfiable, the claim is valid.

3b) “Sophia is either a college professor or a university professor. If Sophia is a college professor, then she has M.S (Master of Science) degree. If Sophia is a university professor and she has a M.S degree, then she is smart. Sophia is not smart, so (logical consequence) she is a college professor.”

Is the argument logically correct? Justify your answer using a Davis-Putnam resolution strategy. Note: To model this problem you must use the following propositions:

P: Sophia is a college professor.

Q: Sophia is a university professor.

R: Sophia has a M.S. degree.

S: Sophia is smart.

Set of Clauses:

F = {{P, Q}, {¬Q, ¬R, S}, {¬S}, {¬P}}

By P:

F = {~~{P, Q}~~, {¬Q, ¬R, S}, {¬S}, ~~{¬P}~~, {Q}}

F = {{¬Q, ¬R, S}, {¬S}, {Q}}

By Q:

F = {~~{¬Q, ¬R, S}~~, {¬S}, ~~{Q}~~, {¬R, S}}

F = {{¬S}, {¬R, S}}

By R:

F = {{¬S}, ~~{¬R, S}~~}

F = {{¬S}}

By S:

F = {~~{¬S}~~}

F = {}

Since the output is the empty set of clauses and the formula is satisfiable, the argument is logically incorrect.